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Invariants in Varieties of Lie Algebras

For a positive integer n, with $n \geq 2$, let L_n be the free Lie algebra over a field K of characteristic 0 and let $P_n = L_n/V_1(L_n)$ and $Q_n = L_n/V_2(L_n)$ be relatively free Lie algebras, with $V_1(L_n) \subseteq V_2(L_n)$. For a non-trivial finite subgroup G of $\mathrm{GL}_n(K)$, let P_n^G and Q_n^G be the Lie subalgebras of invariants in P_n and Q_n, respectively. We give connections between P_n^G and Q_n^G. For $G = S_2$, we apply our methods to L_2/L_2'' and $R_2 = L_2/(\gamma_3(L_2') + (\gamma_3(L_2))')$ (i.e., R_2 is a free (nilpotent of class 2)-by-abelian and abelian-by-(nilpotent of class 2) Lie algebra of rank 2). We give a basis and a minimal infinite generating set for $R_2^{S_2}$ and we find a presentation of $R_2^{S_2}$.

Keywords: Varieties of Lie algebras, relatively free Lie algebras, algebra of invariants, symmetric polynomials, free (nilpotent of class 2)-by-abelian and abelian-by-(nilpotent of class 2) Lie algebra.

MSC: 17B01, 17B30.