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Harmonic Analysis on Inhomogeneous Nilpotent Lie Groups

Let G be a semi-direct product of a normal, vector subgroup by a connected, simply connected nilpotent Lie group. A detailed study of the coadjoint orbits of G in the dual space \mathfrak{g}^* of its Lie algebra \mathfrak{g} is motivated by classical harmonic analysis on solvable Lie groups, culminating in the work of Auslander and Kostant, and by more recent work on generalized continuous wavelets. We apply a procedure for matrix reduction to construct a stratification of the space of coadjoint orbits, where each layer of the stratification has an explicit fiber bundle structure, and provides a criterion for the property of regularity for a coadjoint orbit. Examination of the Zariski open layer Ω_0 then yields an algebraic characterization for regularity, and for both regularity and integrality, of every orbit in Ω_0 . When the criterion for collective regularity holds, we construct a simple and explicit topological cross-section for the coadjoint orbits in Ω_0 . When a criterion fails, then the corresponding property fails for a dense \mathcal{G}_δ set in Ω_0 .

Keywords: Inhomogeneous nilpotent Lie group, semi-direct product, coadjoint orbit.

MSC: 22Exx, 22E25, 22E27.