Journal of Lie Theory 34 (2024) 829–862

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Structure Constants for Simple Lie Algebras from a Principal $\mathfrak{sl}_2$-Triple

For a simple complex Lie algebra $\mathfrak{g}$, fixing a principal $\mathfrak{sl}_2$-triple and highest weight vectors induces a basis of $\mathfrak{g}$ as vector space. For $\mathfrak{sl}_n(\mathbb{C})$, we describe how to compute the Lie bracket in this basis using transvectants. This generalizes a well-known rule for $\mathfrak{sl}_2$ using Poisson brackets and degree 2 monomials in two variables. Our proof method uses a graphical calculus for classical invariant theory. Other Lie algebra types are discussed.

Keywords: Lie algebras, invariant theory, transvectants, 6j-symbols.

MSC: 17B05, 13A50.