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Factorization and Boundedness for Representations of Locally Compact Groups on Topological Vector Spaces

We (a) prove that continuous morphisms from locally compact groups to locally exponential (possibly infinite-dimensional) Lie groups factor through Lie quotients, recovering a result of Shtern's on factoring norm-continuous representations on Banach spaces; (b) characterize the maximal almost-periodicity of the identity component $\mathbb{G}_0 \leq \mathbb{G}$ of a locally compact group in terms of sufficiently discriminating families of continuous functions on \mathbb{G} valued in Hausdorff spaces generalizing an analogous result by Kadison-Singer; (c) apply that characterization to recover the von Neumann kernel of \mathbb{G}_0 as the joint kernel of all appropriately bounded and continuous \mathbb{G} -representations on topological vector spaces extending Kallman's parallel statement for unitary representations, and (d) provide large classes of complete locally convex topological vector spaces (e.g. arbitrary products of Fréchet spaces) with the property that compactgroup representations thereon whose vectors all have finite-dimensional orbits decompose as finite sums of isotypic components. This last result specializes to one of Hofmann-Morris on representations on products of real lines.

Keywords: Lie group, locally compact group, representation, separately continuous, jointly continuous, topological vector space, Banach space, locally convex, absolutely convex, bounded set, barreled, bornological, normable, Banach space, maximally almost periodic,

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