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## Graded Multiplicity in Harmonic Polynomials from the Vinberg Setting

We consider Vinberg  $\theta$ -groups associated to a cyclic quiver on r nodes. Let K be the product of general linear groups associated to the nodes, acting naturally on  $V = \oplus \operatorname{Hom}(V_i, V_{i+1})$ . We study the harmonic polynomials on V in the specific case where dim  $V_i = 2$  for all i. For each multigraded component of the harmonics, we give an explicit decomposition into irreducible representations of K, and additionally describe the multiplicities of each irreducible by counting integral points on certain faces of a polyhedron.

Keywords: Harmonic polynomials, theta-groups, Vinberg pair.

**MSC**: 20G05.