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The Splitters and Equalizers of Triangles

The splitters of a triangle are the lines that bisect its perimeter and the equalizers are those lines that bisect both its perimeter and area. In recent studies, it is proved that a triangle can have either one, two or three equalizers that pass through its incenter. The studies, mainly, concentrate on the existence of the equalizers. Our approach, in this article, is more elementary and algebraic in terms of the side-lengths $c \geq a \geq b$ of $\triangle ABC$ and it provides a comprehensive overview on the equalizers of the triangle. It is based on the fact that a cevian from a vertex and through the *Nagel center* is a splitter. So if, say AA' is a *Nagel* splitter, then a line joining two points, M of $A'C$ and N of AC' is an equalizer if and only if $A'M = AN = x$ and $2x^2+(a+b-3c)x-c(b-c) = 0$. So, by finding all possible solutions, we proved that every triangle can have either one, two or three equalizers, their distribution and locations on the sides are determined, and their geometric construction by compass and ruler is shown. A summary of these results is given in the conclusions section and to make these results more feasible, a visual diagram that predicts the number of equalizers according with the side-length is drawn. For a scalene $\triangle ABC$, we proved that there are no equalizers that cut the smallest two sides, there is only one equalizer cutting the smallest and largest sides, and a maximum of two equalizers that cut the largest two sides.

Keywords: Nagel center, Nagel splitter, cleaver, equalizer.

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