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On Diversities and Finite Dimensional Banach Spaces

A diversity δ in M is a function defined over every finite set of points of M mapped onto $[0,\infty)$, with the properties that $\delta(X)=0$ if and only if $|X|\leq 1$ and $\delta(X\cup Y)\leq\delta(X\cup Z)+\delta(Z\cup Y)$, for every finite sets $X,Y,Z\subset M$ with $|Z|\geq 1$. Its importance relies in the fact that, amongst others, they generalize the notion of metric distance.

We characterize when a diversity δ defined over M, $|M|=3$, is Banach-embeddable, i.e. when there exist points p_i, $i=1,2,3$, and a symmetric, convex, and compact set C such that $\delta(\{x_{i_1},\dots,x_{i_m}\})=R(\{p_{i_1},\dots,p_{i_m}\},C)$, where $R(X,C)$ denotes the circumradius of X with respect to C. Moreover, we also characterize when a diversity δ is a Banach diversity, i.e. when $\delta(X)=R(X,C)$, for every finite set $X\subset\mathbb{R}^n$, where C is an n-dimensional, symmetric, convex, and compact set.

Keywords: Diversity, generalized circumradius, Minkowski diversity, Minkowski embeddable, Banach embeddable.

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