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G. P. Corrêa
Faculty of Exact and Technological Sciences, Federal University of Pará, Abaetetuba, Brazil
genivaldo@ufpa.br

G. C. G. dos Santos
Institute of Exact and Natural Sciences, Federal University of Pará, Belém, Brazil
gelsonsantos@ufpa.br

Schrödinger-Poisson System Involving Potential Vanishing at Infinity and Unbounded Below

This article concerns the following class of system

$$\begin{cases} -\Delta u + V(x)u + \ell(x)\phi u = f(u) + \lambda |u|^{q-2}u & \text{in } \mathbb{R}^3, \\ -\Delta \phi = \ell(x)u^2 & \text{in } \mathbb{R}^3, \\ u, \phi \in D^{1,2}(\mathbb{R}^3),\ u, \phi \geq 0 & \text{in } \mathbb{R}^3, \end{cases}$$

where $\lambda \geq 0$ and $q \geq 2^* = 6$ is the critical Sobolev exponent in dimension 3, the nonlinearity $f : \mathbb{R} \to \mathbb{R}$ is superlinear and has subcritical growth, $V, \ell : \mathbb{R}^3 \to \mathbb{R}$ are measurable functions with $\ell \in L^2(\mathbb{R}^3)$, the potential V can change sign in $\mathbb{R}^3$ and vanish at infinity, that is, $V(x) \to 0$ as $|x| \to \infty$. Our approach is based on variational method combined with Benci-Fortunato's reduction argument [Topol. Methods Nonlinear Anal. 11 (1998) 283–293], Del Pino-Felmer's penalization technique [Calc. Var. Partial Diff. Equations 4 (1996) 121–137] and L^∞-estimate.

Keywords: Schroedinger-Poisson system, variational methods, Mountain Pass Theorem, nontrivial solution, supercritical exponents, vanishing potential, sign-changing potential.

MSC: 35A15, 35Q61, 35B38, 35B09.