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On Strong Convergence for the Split Common Null Point Problem in Two Banach Spaces

Let E be a 2-uniformly convex and uniformly smooth Banach space, F be a smooth, strictly convex and reflexive Banach space and η be a real number with $\eta \in (-\infty, 1)$. Let $T : E \to E$ be a relatively nonexpansive mapping and let $U\colon F \to F$ be an η-demimetric and demiclosed mapping with $F(U) \neq \emptyset$, where $F(U)$ is the set of all fixed points of U. Let $L\colon E \to F$ be a bounded linear operator such that $L \neq 0$ with $F_0 = F(T) \cap L^{-1}F(U) \neq \emptyset$. Then, W. Takahashi [*A strong convergence theorem for solving the split common fixed point problem in two Banach spaces and applications*, Linear and Nonlinear Analysis 6 (2020) 473–495] considered Halpern-type iteration [see B. Halpern, *Fixed points of nonexpanding maps*, Bull. Amer. Math. Soc. 73 (1967) 957–961] and proved a strong convergence theorem for the split common fixed point problem for U and T. From this, we got several new results for the split convex feasibility problem, the split common null point problem and the split common fixed point problem.

In this paper, motivated by the paper of Takahashi cited above, for $p > 1$, when E is a p-uniformly convex and smooth Banach space and F is a strictly convex, reflexive and smooth Banach space, we consider the split common null point problem. We propose another Halpern-type iteration [see K. Nakajo, *Strong convergence for the problem of image recovery by the metric projections in Banach space*, J. Nonlinear Convex Analysis 23 (2022) 357–376] and prove strong convergence theorems, from which we obtain the results for several split problems under weaker conditions than those by Takahashi [loc. cit.].

Keywords: Split common null point problem, split convex feasibility problem, split common fixed point problem, split common equilibrium problem, metric projection.

MSC: 47J25, 47H14.