Journal of Convex Analysis 32 (2025) 961–974

A. Boulkhemair
Jean Leray Math. Lab., UFR-Sciences and Techniques, Nantes, France
boulkhemair-a@univ-nantes.fr

A. Chakib
Applied Mathematics Team, Faculty of Sciences and Techniques, Sultan Moulay Slimane University, Beni Mellal, Morocco
chakib.fstbm@gmail.com

A. Sadik
(1) Jean Leray Math. Lab., UFR-Sciences and Techniques, Nantes, France
(2) Applied Mathematics Team, Faculty of Sciences and Techniques, Sultan Moulay Slimane University, Beni Mellal, Morocco
sadik.fstbm@gmail.com

On Some Uniform Estimates of Gauge Functions with Respect to Domains

We establish uniform estimates and properties of gauge functions for domains Ω_ε, $\varepsilon \in [0,1]$, defined by the Minkowski sum $\Omega_\varepsilon = \Omega_0 + \varepsilon\Omega$ where Ω_0 and Ω are convex and bounded subsets of $\mathbb{R}^n$. These estimates are in fact needed when one deals with shape derivatives in PDE-constrained shape optimization problems using this Minkowski sum as a deformation as it is done in a recent paper of A. Boulkhemair and A. Chakib [*On a shape derivative formula with respect to convex domains*, J. Convex Analysis 21/1 (2014) 67–87] for example. We first show that this class of domains Ω_ε satisfies the so-called uniform ball property which is equivalent to the positiveness of its reach. Then, we establish the said uniform estimates on the gauge function of Ω_ε and its gradient as well as its hessian, with respect to the parameter ε.

Keywords: Convex domains, gauge functions, support functions, Minkowski sum, uniform ball condition, reach condition, uniform estimates.

MSC: 52A07; 52A30, 53A05, 49Q10.