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### **The Gauss Map of a Nonsmooth Convex Cone and the Antipodal Mate Property**

We discuss some aspects concerning the angular structure of a closed convex cone  $K$  in a Euclidean vector space  $E$ . The cone under consideration is assumed to be pointed and solid, but not necessarily smooth. Its Gauss map  $G_K$  is therefore to be understood in a multivalued sense. By definition,  $G_K$  assigns to a boundary point  $u$  of  $K$  the set  $G_K(u) := N_K(u) \cap S_E$ , where  $S_E$  is the unit sphere of  $E$  and  $N_K$  is the normal cone map of  $K$  in the sense of convex analysis. By a positive homogeneity argument, there is no loss of generality in assuming that  $u$  has unit length. Among other issues, we elaborate on the connection between  $G_K(u)$  and the set  $M_K(u)$  of antipodal mates of  $u$ . That  $v$  is an antipodal mate of  $u$  means that  $\{u, v\}$  is a pair of unit vectors in the boundary of  $K$  achieving the maximum angle of the cone.

**Keywords:** Convex cone, maximum angle, largest angle function, oriented distance function, Gauss map, antipodal mate, diametric completeness.

**MSC:** 52A20, 52A40.