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### On the Infimum of the Upper Envelope of Certain Families of Functions

Given a topological space  $X$ , an interval  $I \subseteq \mathbf{R}$  and five continuous functions  $\varphi, \psi, \omega : X \rightarrow \mathbf{R}$ ,  $\alpha, \beta : I \rightarrow \mathbf{R}$ , we are interested in the infimum of the function  $\Phi : X \rightarrow ]-\infty, +\infty]$  defined by

$$\Phi(x) = \sup_{\lambda \in I} (\alpha(\lambda)\varphi(x) + \beta(\lambda)\psi(x)) + \omega(x).$$

Using a recent minimax theorem of the author [see *Minimax theorems in a fully non-convex setting*, J. Nonlinear Var. Analysis 3 (2019) 45-52], we build a general scheme which provides the exact value of  $\inf_X \Phi$  for a large class of functions  $\Phi$ . When additional compactness conditions are satisfied, our scheme provides also the existence of (explicitly detected) functions  $\gamma, \eta : X \rightarrow \mathbf{R}$  such that, for some  $\tilde{x} \in X$ , one has

$$\gamma(\tilde{x})\varphi(\tilde{x}) + \eta(\tilde{x})\psi(\tilde{x}) + \omega(\tilde{x}) = \inf_{x \in X} (\gamma(\tilde{x})\varphi(x) + \eta(\tilde{x})\psi(x) + \omega(x)).$$

**Keywords:** Infimum, minimax, inf-connectdness, inf-compactness.

**MSC:** 49J35, 90C47.