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V. Kryštof
Charles University, Faculty of Mathematics and Physics, Praha, Karlín, Czech Republic
vaaclav.krystof@gmail.com

Functions on a convex set which are both ω-semiconvex and ω-semiconcave II

In a recent article [*Functions on a convex set which are both ω-semiconvex and ω-semiconcave I*, J. Convex Analysis 29 (2022) 837–856] we proved with L. Zajíček that if $G \subset \mathbb{R}^n$ is an unbounded open convex set that does not contain a translation of a convex cone with non-empty interior, then there exist $f : G \to \mathbb{R}$ and a concave modulus ω such that $\lim_{t\to\infty} \omega(t) = \infty$, f is both semiconvex and semiconcave with modulus ω and $f \notin C^{1,\omega}(G)$. Here we improve the previous result as follows: If G is as above and $\omega(t) = t^\alpha$ for some $\alpha \in (0, 1)$, then there exists $f : G \to \mathbb{R}$ that is both semiconvex and semiconcave with modulus ω and $f \notin C^{1,\alpha}(G)$. This result has immediate consequences concerning a first-order quantitative converse Taylor theorem and the problem whether $f \in C^{1,\alpha}(G)$ whenever f is smooth in a corresponding sense on all lines.

Keywords: Semiconvex function with general modulus, semiconcave function with general modulus, $C^{1,\alpha}$ function, $C^{1,\omega}$ function, unbounded open convex set.

MSC: 26B25.