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The Regularity Property of Compensated Convex Transforms for Semiconvex Functions of General Modulus

We establish a general approximations theorem for semiconvex and semiconcave functions with general modulus by using the compensated convex transforms introduced by K. Zhang [*Compensated convexity and its applications*, Ann. Inst. H. Poincaré (C) Non Linear Analysis 25/4 (2008) 743–771]. For a semiconvex function f with general modulus, we show that the limit of the gradient of the upper compensated transform exists and is equal to the center of the minimal bounding sphere in the sense of H. Jung [*Über die kleinste Kugel, die eine räumliche Figur einschließt*, J. Reine Angew. Mathematik 123 (1901) 241–257] of the Fréchet subdifferential. We also prove a C^{1,ω_λ} regularity result for the upper transform of semiconvex functions with general modulus.

Keywords: Compensated convex transforms, convex function, semiconvex function, semiconcave function, linear modulus, general modulus, singularity extraction, minimal bounding sphere, Frechet subdifferential.

MSC: 52A41, 41A30, 26B25, 49J52.