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**Singularities of Fitzpatrick and Convex Functions**

In a pseudo-Euclidean space with scalar product  $S(\cdot, \cdot)$ , we show that the singularities of projections on  $S$ -monotone sets and of the associated Fitzpatrick functions are covered by countable  $c - c$  surfaces having positive normal vectors with respect to the  $S$ -product. By L. Zajíček [*On the differentiation of convex functions in finite and infinite dimensional spaces*, Czechoslovak Math. J. 29/104 (1979) 340–348], the singularities of a convex function  $f$  can be covered by a countable collection of  $c - c$  surfaces. We show that the normal vectors to these surfaces are restricted to the cone generated by  $F - F$ , where  $F := \text{cl range } \nabla f$ , the closure of the range of the gradient of  $f$ .

**Keywords:** Convexity, subdifferential, Fitzpatrick function, projection, pseudo-Euclidean space, normal vector, singularity.

**MSC:** 26B25, 26B05, 47H05, 52A20.