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Stochastic Approximation in Convex Multiobjective Optimization

Given a strictly convex multiobjective optimization problem with objective functions $f_1, \dots, f_N$, let us denote by x_0 its solution, obtained as minimum point of the linear scalarized problem, where the objective function is the convex combination of $f_1, \dots, f_N$ with weights $t_1, \dots, t_N$. The main result of this paper gives an estimation of the averaged error that we make if we approximate x_0 with the minimum point of the convex combinations of n functions, chosen among $f_1, \dots, f_N$, with probabilities $t_1, \dots, t_N$, respectively, and weighted with the same coefficient $1/n$. In particular, we prove that the averaged error considered above converges to 0 as n goes to ∞, uniformly w.r.t. the weights $t_1, \dots, t_N$. The key tool in the proof of our stochastic approximation theorem is a geometrical property, called by us small diameter property, ensuring that the minimum point of a convex combination of the functions $f_1, \dots, f_N$ continuously depends on the coefficients of the convex combination.

Keywords: Multiobjective optimization, continuity of solution map, convex combinations of convex functions, small diameter property.

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