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On Poidge-Convexity

Let $\mathcal{F}$ be a family of sets in $\mathbb{R}^d$ (always $d \geq 2$). A set $M \subset \mathbb{R}^d$ is called $\mathcal{F}$-*convex*, if for any pair of distinct points $x, y \in M$, there is a set $F \in \mathcal{F}$ such that $x, y \in F$ and $F \subset M$. We obtain the poidge-convexity, when $\mathcal{F}$ consists of all unions $\{x\} \cup \sigma$, called *poidges*, where x is a point, σ a line-segment, and $\mathrm{conv}(\{x\} \cup \sigma)$ a right triangle. In this paper we first present several new results on the poidge-convexity of various sets, such as unions of line-segments, fans, cones and cylinders, complements of some given sets and not simply connected sets. Then, we investigate the poidge-convex completion of compact convex sets, trying to determine the minimal number of points necessary to be added to make them poidge-convex.

Keywords: Poidge-convexity, unions of line-segments, complements, poidge-convex completion.

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